Exam 1 Extra-Credit. Due Mon 03/11. CSI 30. Spring 2024. Prof. Pineiro

1. Write the propositions using simpler propositions and logical connectives and determine, if possible, true or false.

   (a) If Philadelphia is the capital of the US then $x < 3$

   (b) If you do not like to come to school then the number 3 is even.

   (c) The water is solid if and only if $4 > 2$.

   (d) 7 is not a prime number or 6 is divisible by 4.

2. State clearly how the expression can be represented using simple expressions and translate: "You cannot enter in the movie if you are under 16 unless you come with your parents."

3. Show that the compound proposition

$$(\neg q \wedge (p \to q)) \to \neg p$$

   is a tautology using a truth table.

4. Show that the compound proposition

$$((p \wedge r) \wedge (p \to q)) \to q \wedge r$$

   is a tautology **without using** a truth table.

5. Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

6. Find **the negation** of each of the following statements.

   (a) Angela is doing this semester Calculus I and Calculus II.

   (b) Angela is doing this semester Calculus I or Calculus II..

   (c) Every student in the class passed the entrance exam. (d) Every student in the class failed at least one of the midterms.

7. Express each of the following statements using predicates, quantifiers, logical connectives, and mathematical operators. Domain: all real numbers.
   (a) 'For every number x and y there is always a number n such that $x(n) > y$'. Domain: x, y real numbers, n a natural number.
   (b) 'Every number have a successor'. Domain: all natural numbers.

   (c) "For all real numbers x and y, there exist a real number z between them." Domain: x, y real numbers.
   (d) "Every positive number is the square of some number". "Domain: Real numbers.

8. Rewrite the statement in such a way that negations appear only within predicates:

$$\neg \forall x \, (\exists y \forall z P(x, y, z) \vee \exists z \forall y Q(x, y, z)).$$

9. Write the converse, inverse and the contrapositive of the statement "If you are a pass all tests and quizzes, then you will know Discrete Mathematics."